Kinetic theory of gases: –

Postulates of Kinetic theory of Gases

(1) All gases are, Composed of a large number of very tiny Particles called molecules. Molecules of a gas are alike but differ from molecules of other gases.

(2) The volume of a gas molecule is negligible in comparison to the total volume of the Gas.

(3) Molecules of a gas are in the state of Constant random motion in all directions and motion increases with increase of temperature.

(4) Molecules of a gas have Kinetic energy only. They do not have Potential energy.

(5) The gas molecules Collide with each other and with the walls of Container.

(6) Gas molecules are rigid and perfectly elastic spheres and exert no force of attraction or repulsion on one another or walls of the Container. each

After collision gas molecules begin to move in the Opposite direction with the same velocity.

(7) The pressure of the gas is due to Collision of molecules with the inner walls of the Container.

(8) The average Kinetic energy of gas molecules is directly proportional to the absolute Temperature.

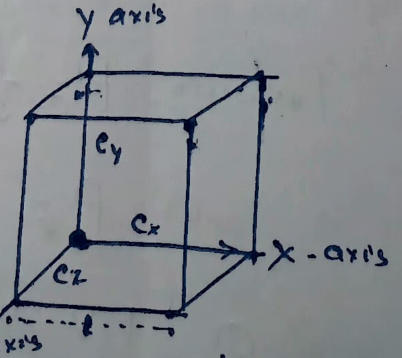
i.e. $K.E \propto T$.

Derivation of Kinetic Gas Equation : –

Let a Cubical Container having length l Contains n molecules of gas having mass m each of them are moving with velocity c.

The velocity of the molecule is c_x along X-direction, c_y along y direction and c_z along z direction respectively

i.e $c^2 = c_x^2 + c_y^2 + c_z^2$



Let us consider the a gas molecule is moving along X-axis and Collide with the wall and back to opposite direction with same velocity c_x, because gas molecule is perfectly elastic sphere.

Now, the change in momentum per Collision along X-axis

$= $ Momentum before Collision – Momentum after Collision

$= mc_x - (-mc_x)$

$s = ut$
$l = c_x \cdot t$
$t = \frac{c_x}{2l}$
Collisions per sec $= \frac{c_x}{2l}$

$\therefore$ Number of Collisions per Second $= \frac{c_x}{2l}$ —— (2)

$\therefore$ Change in momentum along x direction wall per second due to $\frac{c_x}{2l}$ Collisions $= 2mc_x \times \frac{c_x}{2l} = \frac{mc_x^2}{l}$ —— (3)

$\therefore$ Change in momentum per Second due to the collisions of one gas molecule on two Opposite faces along x axis $= \frac{mc_x^2}{l} + \frac{mc_x^2}{l} = \frac{2mc_x^2}{l}$

Therefore the rate of change in momentum due to the collisions per molecule on six faces of the cube $= \frac{2mc_x^2}{l} + \frac{2mc_y^2}{l} + \frac{2mc_z^2}{l}$

$= \frac{2m}{l}(c_x^2 + c_y^2 + c_z^2)$

$= \frac{2mc^2}{l}$

Therefore the rate of change in momentum for n molecules $= \frac{2mnc^2}{l}$ ———— (4)

According to Newton's 2nd law, the rate of change of momentum is equal to force

And we also know that force per unit area is the Pressure of gas

Hence, force $= \frac{2mnc^2}{l}$

and area $= 6l^2$

$\therefore$ Pressure $= \frac{\text{Force}}{\text{Area}} = \frac{2mnc^2}{l \times 6l^2}$

$= \frac{\cancel{2}mnc^2}{3\cancel{6}l^3}$ $[\because l^3 = V]$

$\therefore$ Pressure $P = \frac{1}{3}\frac{mnc^2}{l^3} = \frac{1}{3}\frac{mnc^2}{V}$ —— (5)

or $P = \frac{1}{3}\frac{Mc^2}{V}$ $[\because mn = M]$

or $PV = \frac{1}{3}Mc^2$ ———— (6)

The above equation is Called the Kinetic gas equation.

or, $P = \frac{1}{3}\frac{M}{V}c^2$

$= \frac{1}{3}\rho c^2$

Where ρ is density $(\rho = \frac{M}{V})$

Relation between Kinetic Energy and Temperature of a gas.

The relationship between Kinetic Energy and Temperature can be derived from the Kinetic gas equation.

We know from Kinetic gas equation

$$PV = \frac{1}{3} mnc^2 \quad \text{--- (1)}$$

or $$PV = \frac{2}{3} \times \frac{1}{2} mnc^2$$

or, $$PV = \frac{2}{3} \times \frac{1}{2} Mc^2 \quad \text{--- (2)} \quad \left[\because mn = M, \ M = \text{Mass of the gas}\right]$$

$\because$ $PV = RT$ (For 1 mole of gas) — (3)

From eqn (2) and (3)

$$RT = \frac{2}{3} \times \frac{1}{2} Mc^2$$

$$RT = \frac{2}{3} \times K.E. \quad \left[\because K.E = \frac{1}{2} Mc^2\right]$$

Where M is Mass & c is velocity

$$\therefore K.E = \frac{3}{2} RT$$

or $K.E \propto T$ [Where $\frac{3}{2}R$ is Constant]

So, It is Clear that K.E of translasion of an ideal gas is independent of the nature of the gas and its pressure. It depends only upon the temperature of the gas.

Derivation of the gas-law's on the basis of Kinetic gas equation.

(1) Derivation of the Boyle's Law:-

According to Kinetic theory of gases, the average Kinetic energy ($\frac{1}{2} mnc^2$) is directly proportional to absolute Temperature (T)

i.e. $$\frac{1}{2} mnc^2 = RT$$

$$\frac{3}{2} \times \frac{1}{3} mnc^2 = RT$$

$$\frac{3}{2} P.V = RT \quad \left[\because \frac{1}{3} mnc^2 = PV\right]$$

or $$PV = \frac{2}{3} RT$$

Therefore, the product of Pressure and volume is a Constant at a Constant temperature,